The maximum reduction in resistance for the tube with artificial roughness reached 59% at a metaupone concentration C = 0.6% and Re = 6500, while for the naturally rough tube the maximum reduction was 68% at the same concentration and Re = 12,000. With further increase in velocity there is a quite sharp increase in friction coefficient, i.e., a decrease in the reduction of hydrodynamic resistance. This is evidently related to destruction of the mycelial structures of the surface-active substance in the Reynolds number range above the threshold value.

It follows from the experimental studies performed that addition of polyacrylamide and metaupone decrease hydrodynamic resistance in turbulent flow of liquids in rough tubes: in practice the resistance reduction begins in the region of transition from laminar to turbulent flow: the resistance reduction in metaupone solutions is found over a limited range of change of Reynolds number, which range enlarges with increase in metaupone concentration. Comparison of the results obtained with experimental data for smooth tubes [1, 5] shows that to obtain the same resistance reduction effect a larger concentration of the additive is necessary in rough tubes.

NOTATION

d, inner diameter of tube; R, radius; s, screw pitch; k, height of roughness projection; b_1, distance between projections; h, effective height of projections; k_s, roughness value equivalent to sand roughness; l, length of tube section over which pressure drop was measured; λ, friction resistance coefficient; Re, Reynolds number, calculated from solution viscosity; C, concentration of solute; τ_w, threshold shear stress on tube wall; b, width of roughness projections.

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A GENERALIZED HYDRAULIC RESISTANCE COEFFICIENT

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A generalization of the resistance law for rheologically stationary liquids is considered for flow in tubes and channels of various geometry.

In hydrodynamic calculations the necessity often develops of determining hydraulic losses in motion of liquid in tubes and channels of various cross sections.

The present study will consider the possibility of generalizing the resistance law for rheologically stationary liquids for flow in media of various geometries.

It has been discovered [1] by processing of experimental data on the flow of various non-Newtonian systems that in the case of laminar flow, in the consistent variables chosen, rheometric data for media of various geometries form a single curve. According to Bingham, consistency is defined by complete relationships between force factors and flow characteristics. For the force factor the mean over the perimeter of the shear stress τ_w was chosen, while for the flow characteristic the mean velocity gradient $\dot{\gamma}_w$ was selected. The quantity τ_w is determined from the equilibrium of pressure and friction forces acting on a certain volume of liquid, limited by two sections separated by a distance l, i.e.,

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$$\Delta PF = l \int_{\varkappa} \tau_w(\varkappa)\, ds = l\varkappa \frac{1}{\varkappa} \int_{\varkappa} \tau_w(\varkappa)\, ds = \varkappa l \tau_w,$$

whence

$$\tau_w = \Delta P R_h / l. \tag{1}$$

The choice of the parameter $\dot{\gamma}_w$ was based on the analogy between laminar flow of a liquid and the twisting of a prism of the same cross section (the Boussinesq analogy). An approximate formula for liquid flow rate then has the form

$$Q = \Delta P F^4 / 16\pi^2 I_0 l. \tag{2}$$

After elementary transformations, Eq. (2) becomes

$$\tau_w = \left(\frac{4\pi i_0}{\varkappa}\right)^2 \mu \frac{u_m}{R_h}. \tag{3}$$

From Eq. (3), together with Newton's law, written in the consistent variables τ_w and $\dot{\gamma}_w$, we obtain

$$\dot{\gamma}_w = \left(\frac{4\pi i_0}{\varkappa}\right)^2 \frac{u_m}{R_h} = \xi \frac{u_m}{R_h}. \tag{4}$$

The geometric quantity $\xi = (4\pi i_0/\varkappa)^2$ depends solely on the form of the cross section and is termed the form coefficient. The exact value of ξ may be determined by writing the formulas for flow rates for various cross sections in consistent variables. The values of ξ for media of various geometries are presented in [1].

Thus, a generalized relationship

$$\tau_w = \xi \mu u_m / R_h \tag{5}$$

is obtained.

Equation (5), which relates integral values (pressure drop and flow rate) for media of various geometries, can be considered as applicable to non-Newtonian liquids as well. However, in the case of a non-Newtonian liquid the quantity μ does not remain constant, but is some function of the velocity gradient (shear stress). The form of the function $\mu = f(\dot{\gamma}_w)$ is determined from the condition of best approximation of the actual flow curves. It has been found by processing of experimental data that the generalized model of Shul'man [2] is the most reliable, describing the rheological curves of non-Newtonian liquids over a quite wide range. In consistent variables the Shul'man model is written as

$$\tau_w^{1/n} = \tau_0^{1/n} + (\eta \dot{\gamma}_w)^{1/m}. \tag{6}$$

From Eq. (6) and the Darcy—Weisbachite equation, written in the form

$$\tau_w = \frac{1}{8} \lambda \rho u_m^2, \tag{7}$$

we obtain

$$\lambda = 8 \Bigg/ \left\{ \frac{\rho u_m^{2-i} R_h^i}{\xi^i \eta^i \left[1 + \left(\frac{\tau_0 R_h^i}{\xi^i \eta^i u_m^i}\right)^{1/n}\right]^n} \right\} \quad (i = n/m). \tag{8}$$

If we employ the notation

$$\frac{\rho u_m^{2-i} R_h^i}{\xi^i \eta^i} = \mathrm{Re}', \quad \frac{\tau_0 R_h^i}{\xi^i \eta^i u_m^i} = \Pi, \quad \frac{\mathrm{Re}'}{(1 + \Pi^{1/n})^n} = \mathrm{Re}^*,$$

then from Eq. (8) we have

$$\lambda = 8/\mathrm{Re}^*. \tag{9}$$

As follows from Eq. (9), for laminar flow of a non-Newtonian liquid the criterial equation is written in the form $\lambda = f(\mathrm{Re}', \Pi)$, i.e., there are two similarity criteria: the

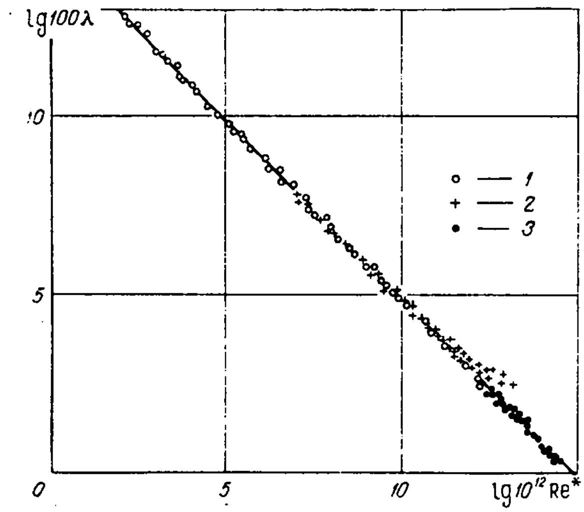

Fig. 1. Hydraulic resistance coefficient versus modified Reynolds number: 1) authors' data on motion of nonlinear viscoplastic liquids in round cylindrical tube, plane slit, and porous medium; 2) Newtonian liquid filtration data [10, 11]; 3) data of Hedstrem, Metzner, and Reed for motion of Bingham and power liquids in round tube [3].

number Re', characterizing the effect of the structural viscosity, and the parameter Π, characterizing the effect of plasticity.

Equation (9) generalizes the resistance law of a large class of rheologically stationary liquids, in particular, liquids following the Hesson model (m = n = 2), Balkly–Herschel model (n = 1), Bingham–Shvedov model (m = n = 1), Ostwald–de Ville model ($\tau_0 = 0$), Newton model ($\tau_0 = 0$, m = n), and others, independent of the geometry of the channel in which the liquid moves. Thus, e.g., for the case of laminar flow of a Newtonian liquid ($\eta = \mu$) in a round tube ($\xi = 2$, $R_h = d/4$), we obtain the well-known relationship

$$\lambda = 8/\left\{\frac{\rho u_m R_h}{\xi\mu}\right\} = 8/\left\{\frac{\rho u_m d}{8\mu}\right\} = \frac{64}{\mathrm{Re}}.$$

It should be noted that Weltman, Metzner, and Reed [3, 4] previously developed universal methods for determining the hydraulic resistance coefficient, applicable to laminar flow of Newtonian and non-Newtonian liquids in a round tube. However, the generalization described above differs in its simplicity, and can be employed for tubes with different cross sections, and also for a porous medium.

Generalization to a porous medium is possible by replacement of the hydraulic radius by the quantity $\sqrt{k}$. Such a substitution was used by Leibenzon, Minskii, and Antonio [5, 6, 7] and evaluated favorably in [8, 9]. In [8] various formulas for determination of λ and Re in filtration of Newtonian liquids were analyzed. In all the formulas (except that of Minskii) the linear measure used was either the effective diameter of the soil particle (a quantity which is difficult to determine and not always exact), or various combinations of the coefficients k and m_1.

The coefficient k is a more universal characteristic of a porous medium, since in laminar filtration it considers the sum of all medium peculiarities, in particular, the medium's porosity, microroughness of individual grains, physicochemical properties of the soil, etc. [6].

The value $\xi = 1$ for a porous medium was established by comparison of Eq. (9) written for a Newtonian liquid with the known Darcy law of subsurface hydraulics.

Thus, in laminar filtration of a non-Newtonian liquid, the following relationships are obtained for λ and Re*:

$$\lambda = 8\sqrt{k}\,\Delta P/\rho u_m^2 l, \tag{10}$$

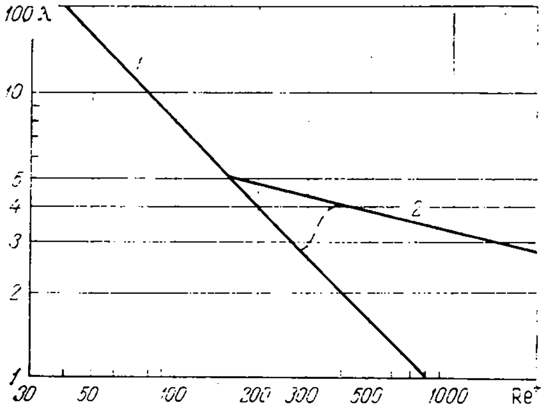

Fig. 2. Generalized function $\lambda = f(\mathrm{Re}^*)$: 1) $\lambda = 8/\mathrm{Re}^*$; 2) $\lambda = 0.1882/\sqrt{\mathrm{Re}^*}$.

TABLE 1. Values of Coefficient λ as Calculated by Empirical Formula of [3] and Eq. (14)

Re*	[3]	Eq. (14)
290	0,01187	0,01140
625	0,00959	0,00939
1250	0,00790	0,00796
3750	0,00600	0,00602
7500	0,00505	0,00510
12500	0,00454	0,00444

$$\mathrm{Re}^* = \rho u_m^{2-i} k^{i/2}/\eta^i \left[1 + \left(\frac{\tau_0 k^{i/2}}{\eta^i u_m^i}\right)^{1/n}\right]^n. \tag{11}$$

For the case of filtration of a Newtonian liquid, the number Re is determined by the formula

$$\mathrm{Re} = \rho u_m \sqrt{k}/\mu. \tag{12}$$

Equation (10) differs from Minskii's formula in the value of the constant.

To verify the applicability of the above generalization, experiments were performed with various non-Newtonian liquids (lubricating oil, a lubricating oil-bright stock mixture, and petroleum samples of various origins from Azerbaidzhan) in models of a round cylindrical tube, a porous medium, and a plane slit. Results are shown in Fig. 1. Also shown there are experimental data of Abdulvagabov [10] and Trebin [11] on filtration of Newtonian liquids, and data of Hedstrem, Metzner, and Reed [3] for flow of Bingham and power liquids in a tube.

Thus, the proposed generalization makes possible determination of hydraulic losses in the laminar flow regime for any rheologically stationary liquid in media of various geometries (including porous media).

It has been established that the critical value of Reynolds number Re^* for motion of rheologically stationary liquids in tubes of various geometries (exluding a porous medium) is approximately equal to 290.

An attempt was made to generalize the resistance law for rheologically stationary liquids in turbulent flow by replacement of the actual velocity profile by a parabolic one (the Prandtl method).

The resistance law for turbulent flow of a Newtonian liquid in a round tube then has the form

$$\lambda = 2^{\frac{n-1}{n-1}} [(n+1)(n+2)]^{\frac{2}{n+1}} / B^{\frac{2}{n+1}} \left(\frac{\rho u_m d}{8\mu}\right)^{\frac{2n}{n+1}}. \quad (13)$$

The experimental studies of Schiller and Nikuradze [12, 13] show that Eq. (13) may be generalized to tubes of noncircular cross section by substitution of the hydraulic radius for the conventional one. Moreover, as in the case of laminar flow, upon replacement of μ by the effective viscosity Eq. (13) may also be used for non-Newtonian liquids. Assuming the validity of the Blasius law ($n = {}^1/_7$, B = 8.74), from Eq. (13) we obtain the approximate relationship

$$\lambda = 0.1882 \sqrt[4]{Re^*}. \quad (14)$$

Equation (14) is generalized to the case of turbulent flow of any rheologically stationary liquid in media of various cross sections for Reynolds number variation over the range Re* = 290-12,500.

To verify the applicability of Eq. (14) for non-Newtonian liquids, the experimental data of Metzner and Reed [3] were processed, with the results shown in Table 1. As is evident from the table, the λ values determined by Eq. (14) and the empirical formula of [3] differ insignificantly (by not more than 4%).

The generalized function λ = f(Re*) for rheologically stationary liquids is shown in Fig. 2.

It should be noted that resistance law (14) should not be used for nonsmooth tubes and porous media, since in turbulent motion the coefficient λ will depend both on Re* and on relative roughness of the medium.

In [14] a similar resistance law analogous to the Blasius formula, obtained by Minigazimov (formula III.6) was presented. That formula was obtained by processing of experimental data on turbulent flow of non-Newtonian systems. In his formula the normal viscosity in the Reynolds number is replaced by a viscosity obtained in the Poiseuille portion of the rheological curve.

NOTATION

τ_w, mean shear stress over perimeter; $\dot{\gamma}_w$, mean velocity gradient; ΔP, pressure drop; l, longitudinal dimension; F, cross-sectional area; $\tau_w(\varkappa)$, shear stress, variable over perimeter; $\varkappa$, wetted perimeter of section; R_h, hydraulic radius; ds, differential perimeter arc; Q, flow rate of liquid; I_0, polar moment of inertia; i_0, polar radius of inertia; μ, dynamic viscosity of Newtonian liquid; u_m, mean velocity (filtration rate); ξ, form coefficient; τ_0, limiting shear stress; η, structural viscosity; m, n, exponents; λ, hydraulic resistance coefficient; Re, Reynolds number for motion (filtration) of Newtonian liquid; Re', generalized Reynolds number for Newtonian liquids; Re*, modified Reynolds number for rheologically stationary liquids; Π, generalized plasticity parameter; ρ, density; B, abstract number; k, permeability coefficient; m_1, porosity.

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INTENSIFICATION OF CONVECTIVE HEAT EXCHANGE IN ANOMALOUSLY VISCOUS MEDIA BY THE APPLICATION OF ARTIFICIAL PERIODIC ROUGHNESS

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The results of an experimental investigation of the intensification of convective heat exchange in the flow of anomalously viscous liquids in pipes with artificial periodic roughness are presented. An estimate is made of the thermohydrodynamic efficiency of the application of this means of intensification to anomalously viscous media.

One of the most critical problems facing industry is to increase the unit output and productivity of equipment, particularly heat exchangers. The most effective way of solving this problem is to develop and investigate methods for the intensification of convective heat exchange.

A well-known means of intensification of heat exchange is to use pipes having an artificial periodic roughness in the form of different kinds of projections or diaphragms on the inner surface of the pipe as the working channels of the heat-exchange apparatus. However, all the test data available in the literature on the intensification of heat exchange in this way pertain to the case of flow of viscous liquids in channels [1-5]. In this connection, it is known [1] that the given method intensifies the heat exchange in viscous liquids by an average of 2.5 times. It is also known [6] that a change in the shape of the profile of a projection when the spacing and height are unchanged has a weak effect on the change in heat transfer while it affects to a considerably greater extent the change in the coefficient of hydraulic resistance, which decreases in proportion to the decrease in the coefficient of profile drag. For example, the lowest hydraulic losses, at a practically equal gain in heat transfer, are achieved in the case of smoothly profiled diaphragms by rolling the outer surface of pipes with rollers. The technology for making such pipes is very simple and the cost of the rolling is a few percent of the cost of a smooth pipe [1]. Moreover, it becomes possible to intensify heat transfer to the outer surface of the pipe also.

Unfortunately, experimental data on the intensification of heat exchange by the indicated means are presently absent for the flow of anomalously viscous liquids.

Since the use of pipes with artificial periodic roughness in the form of rolled smoothly profiled diaphragms yields a considerable gain in heat transfer in the flow of viscous

TABLE 1. Thermophysical Characteristics of Polymer Solutions

Aqueous solution	ρ, kg/m^3	$c_p \cdot 10^{-3}$, J/kg·deg	λ, W/m·deg
PVA, 9%	1020	3,607	0,674
Na CMC, 8.5%	1080	3,289	0,476

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